Torque and Equilibrium

In the notes "bodies in Equilibrium", it was stated that static equilibrium can only truly exist if the net force is zero and if there is no net torque. In particular, if no rotation is to occur, the clockwise and counterclockwise torques must cancel each other out.

Since no rotation occurs, ANY POINT may be selected to be the pivot about which torques are calculated. Usually the pivot is selected to be where some unknown force acts. This effectively eliminates that force from the calculation, and simplifies the problem (remember, forces have no 'torque effect' at the pivot).

For non-concurrent forces, if no translational and no rotational motion is to occur, both conditions of equilibrium must be met: $\tau_{Net} = 0$, $F_{Net} = 0$

(1) sum of the y axis forces = 0 → $\Sigma y = 0$

sum of the x axis forces = 0 → $\Sigma x = 0$

(2) sum of the torques = 0 → $\Sigma \tau = 0$

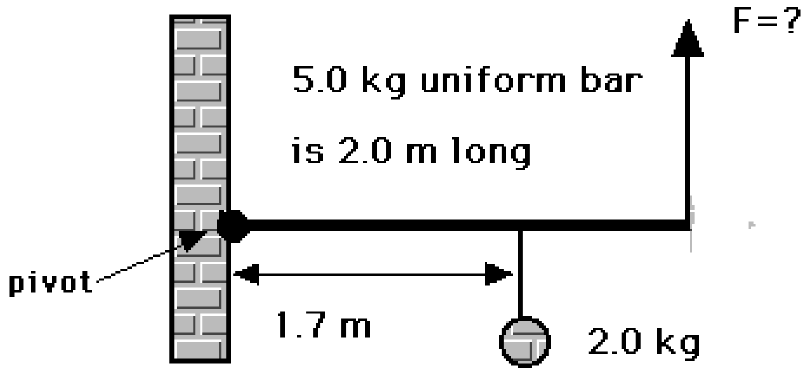
→ **that is,** $\Sigma \tau_{cw} = \Sigma \tau_{ccw}$

When a system is in equilibrium, and forces are non-concurrent, follow these steps to solve:

1. Determine as many forces as you can that act in the diagram. This may involve finding the weight of various masses, or using $F_{Net} = 0$ to analyze unknown forces.
2. Choose a pivot where unknown and un-wanted forces are acting, to eliminate them from any torque calculations.
3. Determine the distances away from the pivot for each remaining force, including the centre-of-gravity force for the beam.
4. For any forces not perpendicular to the length of the beam (or the distance away from the pivot), use trig to find the perpendicular component for each force.

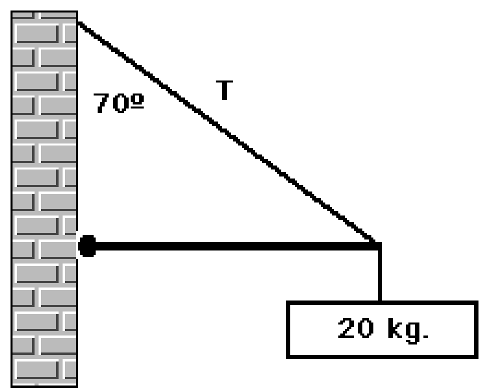
For the following four examples (11-15), the pivot should be located where the beam is attached to the wall or floor, so the frictional and normal forces acting on the beam can be ignored in your torque calculations.

Example #11. A 5.0 kg uniform bar is attached to the wall as shown below, with a 2.0 kg weight hung in the indicated location. What minimum vertical force is needed to cause the system to be in rotational equilibrium?



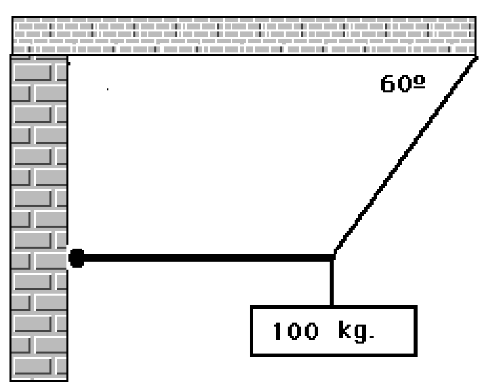
(see Equilibrium Ex 11 for answer)

Example #12. In the following diagram, the 1.6 m-long uniform bar has a mass of 5.0 kg. Calculate the tension "T" of the cable supporting the 20 kg mass hanging on the end of the beam.



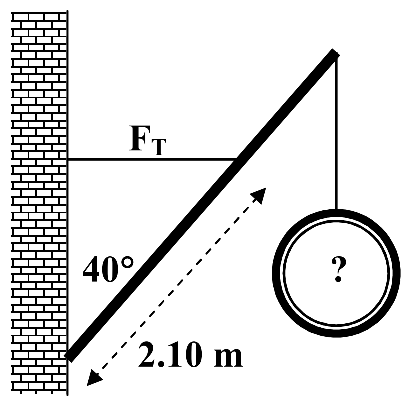
(see Equilibrium Ex 12 for answer)

Example #13. A 8.00 kg uniform beam of length 3.00 m is attached to a wall by a hinge and is supported from the ceiling by a rope which makes an angle of 60° with the horizontal, as shown below. Calculate the tension in the rope that supports the beam.



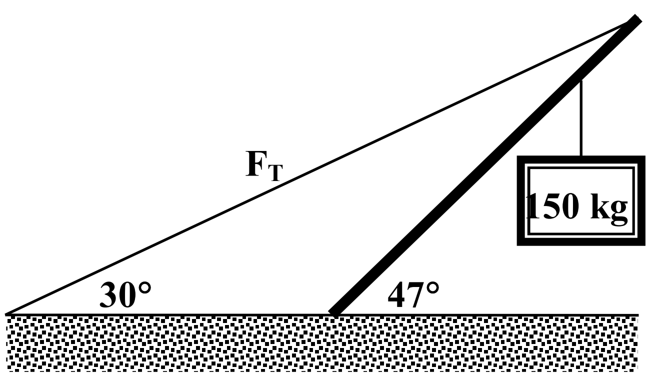
(see Equilibrium Ex 13 for answer)

Example #14. The 12.0 kg uniform boom below has a length of 3.40 m. The cable can withstand a tension 'F_T' of 1850 N before breaking. What is the largest weight that can be hung from the boom in the location indicated?



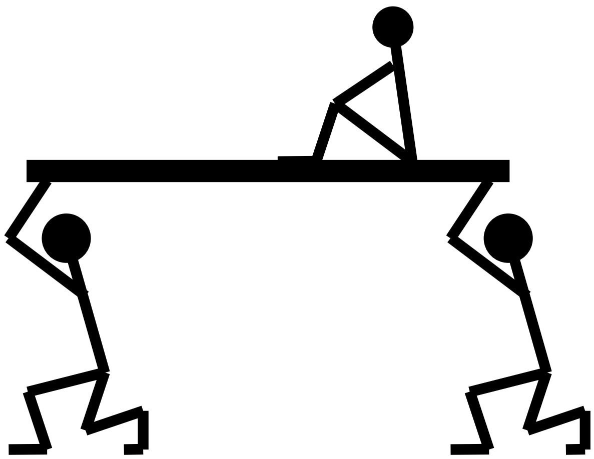
(see Equilibrium Ex 14 for answer)

Example #15. In the diagram below, the uniform boom has a mass of 25.0 kg and a total length of 6.50 m. If the 150 kg mass hangs 4.75 m from where the boom is anchored to the ground, how much tension 'F_T' is in the cable that supports this system?



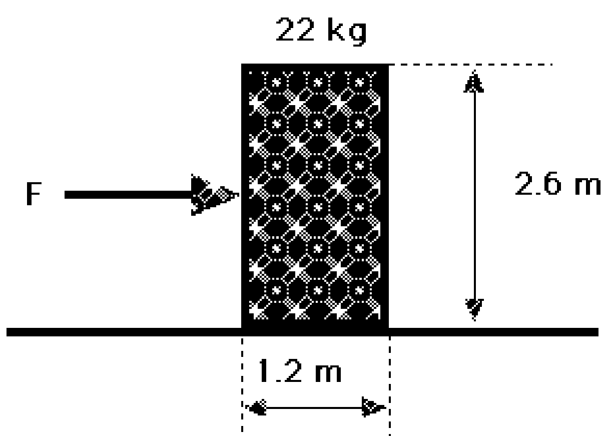
(see Equilibrium Ex 15 for answer)

Example #16. Two students, Phreddie and Phreida Physics, are carrying Normie Neutron on each end of a 20.4 kg uniform plank that is 3.00 m long. If Normie's mass is 51.0 kg and he is sitting 1.00 m from Phreida, how much lifting force does each student use to carry Normie?
Hint: take the pivot at one end in order to remove the force supplied by one student from the situation.



(see Equilibrium Ex 16 for answer)

Example #17. What force, applied half way up the block, will <u>just</u> start the 22 kg block tipping?



(see Equilibrium Ex 17 for answer)

For this final question, you will need to consider both aspects of static equilibrium, that is,

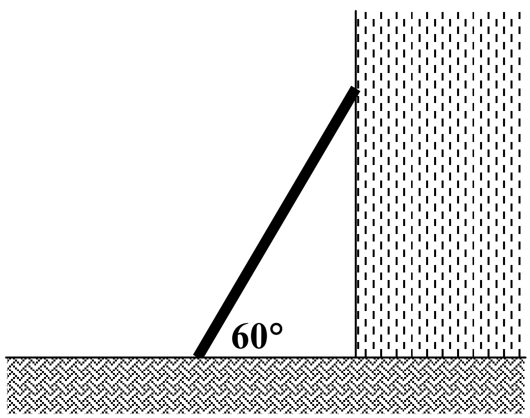
- $\mathbf{F_{Net} = 0}$ in both the horizontal and vertical direction;
- $\mathbf{\Sigma\ \tau_{cw} = \Sigma\ \tau_{ccw}}$

Start by examining a free-body diagram of all the forces acting on the ladder. Then choose your pivot for the ladder at the floor, to eliminate the two forces acting there.

Example #18. A uniform 20.0 kg, 5.00 m-long ladder leans against a smooth (frictionless) wall as shown. Find:

a) the normal force of the floor pushing up against the ladder;
b) the normal force of the wall pushing against the ladder;
c) the friction force between the ladder and the floor;
d) the overall force that the floor exerts on the ladder.

Note: A frictionless surface can only exert a normal force – i.e., a force in a direction that is <u>perpendicular</u> to its surface.



(see Equilibrium Ex 18 for answer)